Section 8.2

Representation of Matrices

a) A matrix can be denoted by an uppercase letter such as A, B, or C.

b) A matrix can be denoted by a representative element enclosed in brackets, such as $[a_{ij}], [b_{ij}]$, or $[c_{ij}]$.

c) A matrix can be denoted by a rectangular array of numbers.

Equality of Matrices

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are equal if they have the same order $(m \times n)$ and $a_{ij} = b_{ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$.

Matrix Addition

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are matrices of order $m \times n$, their sum is the $m \times n$ matrix given by $A + B = [a_{ij} + b_{ij}]$.

Scalar Multiplication

If $A = [a_{ij}]$ is an $m \times n$ matrix and c is a scalar, the scalar multiple of A by c is the $m \times n$ matrix given by $cA = [ca_{ij}]$.

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ matrices and let c and d be scalars.

a) $A + B = B + A$, commutative property.

b) $A + (B + C) = (A + B) + C$, associative property.

c) $(cd)A = c(dA)$

d) $c(A + B) = cA + cB$, $(c + d)A = cA + dA$, distributive property.

Matrix Multiplication

If $A = [a_{ij}]$ is an $m \times n$ matrix and $B = [b_{ij}]$ is an $n \times p$ matrix, the product AB is an $m \times p$ matrix $AB = [c_{ij}]$, where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj}$.

Problem 1. Find x and y.

a) $\begin{bmatrix} 2 & x \\ y & -6 \end{bmatrix} = \begin{bmatrix} 2 & -7 \\ 10 & -6 \end{bmatrix}$

b) $\begin{bmatrix} x-4 & 3 & -8 \\ -1 & 3y & 5x \\ 4 & -6 & y-2 \end{bmatrix} = \begin{bmatrix} 2x-1 & 3 & -8 \\ -1 & 12 & -15 \\ 4 & -6 & 2 \end{bmatrix}$

Problem 2. If possible, find $A + B$, and $3A - 2B$.

a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 4 \\ 5 & -3 \end{bmatrix}$

b) $A = \begin{bmatrix} -2 & 5 \\ 3 & -1 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 6 & -4 \\ 1 & -2 \end{bmatrix}$

c) $A = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -6 & 5 \end{bmatrix}$

Problem 3. Solve for X in the equation $2A + 4B = -2X$, given $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ -5 & 1 \end{bmatrix}$

Problem 4. If possible, find AB.

a) $A = \begin{bmatrix} 2 & -4 & 1 & 0 \\ 3 & 1 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

b) $A = \begin{bmatrix} -2 & 1 \\ 0 & 4 \\ -3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 4 & 5 \end{bmatrix}$

c) $A = \begin{bmatrix} 0 & 2 & -1 \\ 5 & 4 & -3 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -4 & 6 \\ 1 & 3 & -2 \end{bmatrix}$

Problem 5. Find AB, BA, A^2, and X.

a) $A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$. $X = BA - AB - A^2$

b) $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$. $X = A^2 - AB - BA$